2.2 Normal Distributions

HW: p. 131 (41-59 odd, 63, 65, 66, 68-74)

Normal Distributions

We _____ Normal to remind you that these curves are special.

The family of _____ curves share some characteristics that help us perform _____ about observations in a distribution of data:

- They have the same overall shape:
- The mean is at the _____ of the curve.
- Almost all of the observations in an approximately Normal distribution will fall within _____ standard deviations of the _____.

We abbreviate as $N(\quad , \quad)$
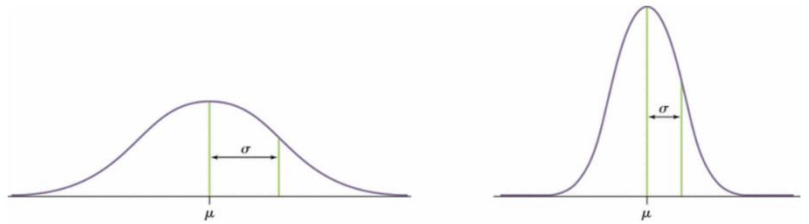
To sketch a Normal curve:

1. Draw a number line. Mark the _____ μ.
2. Draw _____ tick marks to the _____ of the center and three tick marks to the _____ (all evenly spaced). Label them $\mu \pm \sigma, \ \mu \pm 2\sigma, \ \mu \pm 3\sigma$
3. Draw a dot above the mean where the _____ of the curve will be.
4. Above the marks ____ standard deviation from the mean, draw dots about _____% as high as the dot above the mean.
5. Above the marks ____ standard deviations from the mean, draw dots about _____-_____ as high as the previous dots.
6. Above the marks _____ standard deviations from the mean, draw dots _____ _____ the horizontal axis.
7. Connect the seven dots with a _____ curve.
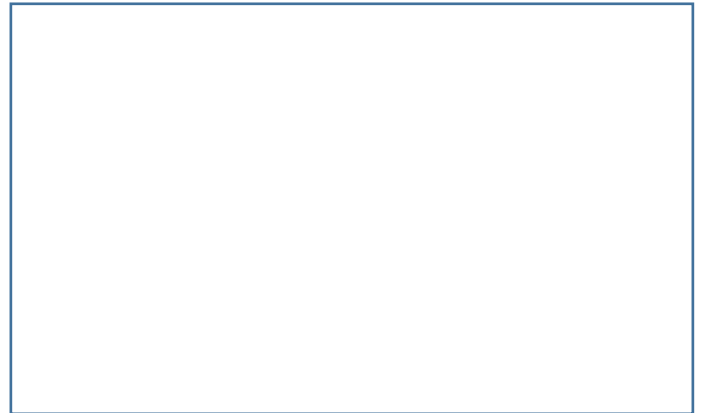
## The 68-95-99.7 Rule (Empirical Rule)

In the Normal distribution with mean μ and standard deviation σ:

- Approximately _____% of the observations will fall within σ of the mean μ.
- Approximately _____% of the observations will fall within 2σ of μ.
- Approximately _____% of the observations will fall within 3σ of μ.

This allows us to perform calculations about the _____ proportion of observations in a distribution that fall into certain _____ without even knowing all of the individual observations.
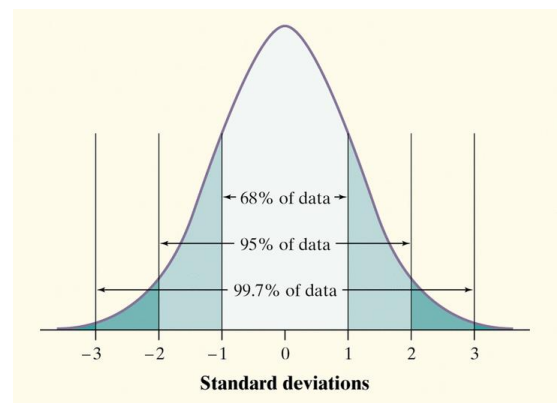
Check Your Understanding, *p. 114*

The distribution of heights of young women aged 18 to 24 is approximately N(64.5, 2.5).
1. Sketch a Normal density curve for the distribution of young women's heights. Label the points one, two, and three standard deviations from the mean.
2. What percent of young women have heights greater than 67 inches? Show your work.

3. What percent of young women have heights between 62 and 72 inches? Show your work.

Example:

According to the CDC, the heights of 12-year-old males are approximately Normally distributed with a mean of 149 cm and a standard deviation of 9 cm.
1. Sketch the distribution, labeling the means and the points one, two, and three standard deviations from the mean.
2. About what percentage of 12-year-old boys will be over 158 cm tall?

3. About what percentage will be between 131 and 140 cm tall?

4. Suppose that a distribution of test scores is approximately Normal and the middle 95% of scores are between 72 and 84. What are the mean and standard deviation of this distribution?

5. Can you calculate the percent of scores that are above 80? Explain.
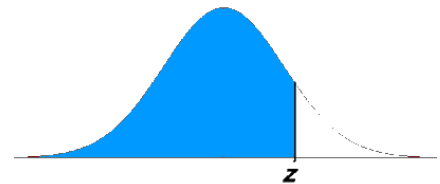
The Standard Normal Distribution

- The _____ Normal distribution is the Normal distribution with mean _ and standard deviation _.
- In practice, ____ ____ observations of interest will fall one, two, or three standard deviations from the mean.
- In cases like this, the 68-95-99.7 rule can help us _____ the proportion, but we'll want to be more _____.
- We can use the standard Normal table and Normal calculations to help us find the proportion.
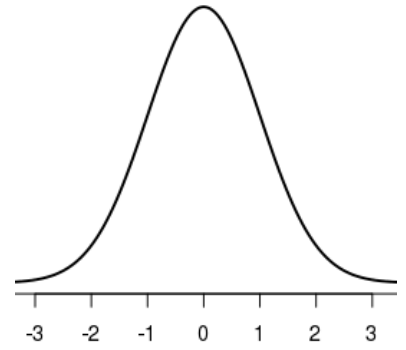
The Standard Normal Table

The Standard Normal Distribution Table is a table of _____ under the standard Normal curve.

The table entry for each value z is the area under the curve to the _____ of z.

Check Your Understanding, *p. 119*

Use the standard Normal table to find the proportion of observations from a standard Normal distribution that fall in each of the following regions. In each case, sketch a standard Normal curve and shade the area representing the region.

1. $z < 1.39$

2. $z > 2.15$

Use the standard Normal table to find the value z from the standard Normal distribution that satisfies each of the following conditions. In each case, sketch a standard Normal curve with your value of z marked on the axis.

3. The 20th percentile

4. 45% of all observations are greater than z.

Normal Distribution Calculations

We can answer a question about proportions of *any* Normal distribution by standardizing and then using the standard Normal table.

Follow the four-step process:

1. STATE: Express the _____ in terms of a variable x.
2. PLAN: _____ a picture of the distribution and _____ the area of interest.
3. DO: Perform _____ by standardizing x and then using Table A or your calculator to find the required area under the Normal curve.
4. CONCLUDE: Write your _____ in the _____ of the problem.

Example: *Serving Speed*
In the 2008 Wimbledon tennis tournament, Rafael Nadal averaged 115 miles per hour (mph) on his first serves. Assume that the distribution of his first serve speeds is Normal with a mean of 115 and a standard deviation of 6 mph.

    a. About what proportion of his first serves would you expect to exceed 120 mph?

    b. What percent of Rafael Nadal's first serves are between 100 and 110 mph?

    c. The fastest 30% of Nadal's first serves go at least what speed?

    d. What is the IQR for the distribution of Nadal's first serve speeds?

    e. A different player has a standard deviation of 8 mph on his first serves and 20% of his serves go less than 100 mph. If the distribution of his serve speeds is approximately Normal what is his average first serve speed?

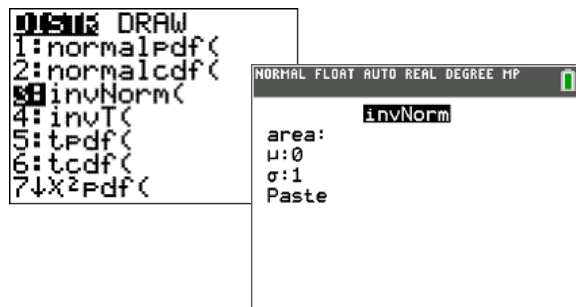## Normal Calculations on Your Calculator

Finding Areas:

- 2nd → vars

- 2:normalcdf(

- Fill in information:
  lower: left boundary
  upper: right boundary
  μ: mean
  σ: standard deviation
  Select Paste
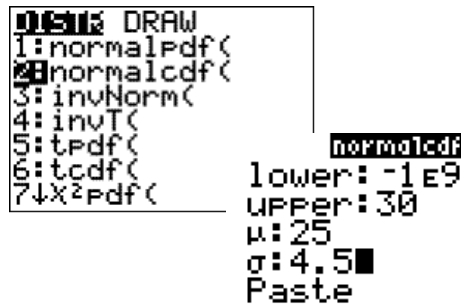
Finding Boundaries (when you know the area)

- 2nd → vars

- 3:invNorm(

- Fill in the following:
  Area: area to the left of the boundary
  μ:
  σ:
  Select Paste

Example:
Suppose that Zack Greinke of the Kansas City Royals throws his fastball with a mean velocity of 94 miles per hour and a standard deviation of 2 mph and that the distribution of his fastball speeds can be modeled by a Normal distribution.

    a) About what proportion of his fastballs with travel over 100 mph?

    b) About what proportion of his fastballs will travel less than 90 mph?

    c) About what proportion of his fastballs will travel between 93 and 95 mph?

    d) What is the 30th percentile of Greinke's distribution of fastball velocities?

    e) What fastball velocities would be considered low outliers for Zach Greinke?

    f) Suppose that a different pitcher's fastballs have a mean velocity of 92 mph and 40% of his fastballs go less than 90 mph. What is his standard deviation of his fastball velocities, assuming his distribution of velocities can be modeled by a Normal distribution?

Assessing Normality

Just because a distribution is symmetric, single-peaked, and bell-shaped, it is _____ necessarily Normal.

_____ performing Normal calculations, you should ___ _____ the distribution you are exploring really is _____.

There are two methods to check for Normality:

*
*

68-95-99.7 Rule for Assessing Normality
1. _____ the _____. Make a dotplot, stemplot, or histogram. See if the graph is approximately _____ and _____-shaped.
2. _____ whether the data follow the 68-95-99.7 rule. _____ the number of _____ within one, two, and three standard deviations of the mean.

Probability Plots for Assessing Normality
- Enter the data values in list 1
  - Stat
  - 1:Edit
  - Enter values
- Define Plot1 as shown to the right ($2^{nd} \rightarrow y =$)
- Zoom
  - 9:ZoomStat

*If the points on the Normal probability plot lie close to a _____ _____, the plot indicates that the data are Normal.

**Systematic _____ from a straight _____ indicate _____-Normal distribution.

*** _____ appear as points that are far _____ from the overall _____ of the plot.

Example: *Quiz Scores*
Below are quiz scores. Construct a Normal probability plot to determine whether these data are approximately Normally distributed.

```
0 | 0 1 2
1 | 2 2 4 8
2 | 1 1 3 4 5 9 9
3 | 0 0 0 3 6
4 | 4 5 7
5 | 0
```